FIRE CODE REFORM

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The Probability of Death in the Room of Fire Origin: an Engineering Formula

FCRC Project 4
Fire Safety System Design Solutions
Part A – Core Model & Residential Buildings

Fire Code Reform Research Program
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Background

The Fire Code Reform Research Program is funded by voluntary contributions from regulatory authorities, research organisations and industry participants.

Project 4 of the Program involved development of a Fundamental Model, incorporating fire-engineering, risk-assessment methodology and study of human behaviour in order to predict the performance of building fire safety system designs in terms of Expected Risk to Life (ERL) and Fire Cost Expectation (FCE). Part 1 of the project relates to Residential Buildings as defined in Classes 2 to 4 of the Building Code of Australia.

This Report was relevant to the project activities in support of the Model's development and it is published in order to disseminate the information it contains more widely to the building fire safety community.

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Comments

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THE PROBABILITY OF DEATH IN THE ROOM OF FIRE ORIGIN: AN ENGINEERING FORMULA

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1 The model

Consider a particular occupant of the room. When a fire starts it may take a short or a long time until it causes a cue to be given to that occupant.This may be an alarm, smoke, flame, window glass breaking, or the arrival of the fire brigade. To each of those cues is attached a probability that it will be the first to alert the considered occupant. For the purposes of the present model, we shall assume that there is only one cue. More than one cue can be accomodated as follows: the conditional probability of death of the considered occupant is calculated for the case of each of the considered cues being the first to alert the occupant. Then the obtained values are combined, using the theorem of total probability, from the knowledge of the probabilities attached to the various cues.

Starting from the point of time when the alerting cue occurs, we denote by X the random time until the onset of "untenable conditions" in the room of fire origin. By "untenable conditions" we mean a room condition which will result in the death of the considered occupant if they have not evacuated by then.The distribution of X can be estimated by running a Monte Carlo simulation, using the appropriate program for compartment fires and monitoring the cumulative effect of toxic fumes and radiation on the considered occupant. The stochastic variation is introduced by sampling the parameters of the fire program from suitable probability distributions. To take into account the fact that X cannot take negative values, it is convenient to write $\ln X = U$, and to work with U rather than X. In view of the impossibility of having accurate knowledge of the conditions that will prevail in an actual fire, it is not unreasonable, for engineering design purposes, to subsume our knowledge of X by just two parameters:

$$\mathrm{E}(U) = \mu_U \tag{1}$$

$$\mathrm{Var}(U) = \sigma_U^2. \tag{2}$$

Also starting from the point of time when the alerting cue occurs, we denote by Y the time until the considered occupant evacuates the room. This is made up of a number of components: a waking up time component (mainly at night), a cognitive process time component, an investigation component and an evacuation preparation component. The distribution of Y should be derived from statistical surveys. For the purpose of engineering design, we lump together all these components, write ln Y = V and subsume our knowledge of Y by just two parameters:

$$\mathrm{E}(V) = \mu_V \tag{3}$$
$$\mathrm{Var}(V) = \sigma_V^2. \tag{4}$$

The basic assumption of the model is, as already noted above, that the considered occupant will incur death if and only if their evacuation time Y is longer than the time of onset of untenable conditions X.

2 Calculation of a safety index

It follows from the basic assumption of the model that the event of the considered occupant not dying is identical to the event X > Y. But in view of the monotonicity of the logarithmic transformation this event is identical to the event $U > V$.

We now need to address the problem of the dependence between X and Y. It seems reasonable to assume that if the fire is severe untenable conditions will occur early. At the same time, the fire will supply more compelling cues to the considered occupant, so that we can assume that evacuation time will tend to be shorter. We conclude that there will be some positive correlation between X and Y, and therefore also between U and V. Let the correlation between U and V be denoted by ρ.

Let now $W = V - U$, E(W) = μ_W, Var(W) = σ_W^2. Then

$$\mu_W = \mu_V - \mu_U \tag{5}$$
$$\sigma_W^2 = \sigma_V^2 + \sigma_U^2 - 2\rho\sigma_V\sigma_U. \tag{6}$$

The safety index β (see Melchers [1] p.37) is given by $\beta = \mu_W/\sigma_W$.

Thus, in order to achieve a safety index of β, it is necessary to choose the parameters of the onset of untenable conditions in such a way that

$$\mu_V > \mu_U + \sqrt{\quad \sigma_V^2 - 2\rho\sigma_U\sigma_V}. \tag{7}$$

The appropriate value of the safety index depends on the severity of the risk to life and property. For example, it would be appropriate to take β = 2.19 if there are 10 occupants at risk but β = 2.63 if there *are* 100 occupants at risk. The rationale for these recommendations will be given in Section 4.

3 Simplifications.

The above result can be greatly simplified if we are prepared to make some conservative assumptions.

Firstly, it should be noted that there is very scant knowledge of the value of the correlation ρ in real situations. Fortunately, since, as pointed out above, ρ is positive, it is a conservative assumption to assume it to be zero, as can be seen from equation (7).

Secondly, suppose that we define two "design values" for U and V as follows:

$$U_{des} = \mu_U - \beta^\star \sigma_U, \quad (8)$$
$$\textit{ties} = \mu_V + \beta^\star \sigma_V. \quad (9)$$

We choose $\beta^\star$ in such a way that when V_{des} is less than U_{des} the safety index is greater than β. Then it is easy to see that the appropriate value of $\beta^\star$ is given by

$$\beta^\star = \beta \frac{\sqrt{\sigma_U^2 + \sigma_V^2}}{\sigma_U + \sigma_V}. \quad (10)$$

From this it follows that $\beta^\star$ will vary from 0.707β (when-the two standard deviations are equal) to β (when one of the standard deviations is zero. Thus, it is again a conservative assumption to take $\beta^\star = \beta$.

Let us now estimate "characteristic values" for U and V as follows:

$$U_C = \mu_U - k\sigma_U, \quad (11)$$
$$\mathrm{VC} = \mu_V + k\sigma_V \quad (12)$$

where k is an appropriate value which depends on the reliability of our estimation. Recommended values are given in Section 4.

It then turns out that

$$U_{des} = U_C - (\beta^\star - k)\sigma_U, \quad (13)$$
$$V_{des} = (\beta^\star - k)\sigma_V. \quad (14)$$

Finally, we can revert to the original values of the variables X and Y as follows: Write

$$X_C = \exp(U_C) \quad (15)$$
$$Y_C = \exp(V_C). \quad (16)$$

Furthermore, let

$$SF_X = \exp[(\beta^\star - k)\sigma_U] \quad (17)$$
$$SF_Y = \exp[(\beta^\star - k)\sigma_V]. \quad (18)$$

Note that X_C and Y_C are simply the quantiles of X and Y corresponding to U_C and V_C. As for SF_X and SF_Y, they are the partial safety factors corresponding to the variables X and Y respectively.

The design values for X and Y, X_{des} and Y_{des}, will then be given by the standard formulae:

$$X_{des} = X_C / SF_X \quad (19)$$
$$Y_{des} = Y_C . SF_Y. \quad (20)$$

Thus, in each design class, the partial safety factors can be calculated in terms of the standard deviations of U and V.

4 Distributional assumptions.

In the previous sections no distributional assumptions have been made and attention has been focussed on means, standard deviations and correlations.

While this approach provides a convenient way to compare the safety of different designs, it does not yield *actual* probabilities of death. To obtain the latter, it is unavoidable to make distributional assumptions.

A very convenient and robust assumption when dealing with non-negative variables such as X and Y above is to assume that they have a lognormal distribution. This of course implies that U and V are normally distributed.

The rationale behind the recommended values of β in Section 2 becomes then apparent. is normally distributed the probability of death for each occupant corresponding. to $\beta = 2.19$ is 0.001 and the expected number of deaths in one fire among 10 occupants is 0.01. Similarly the probability of death for each occupant corresponding to $\beta = 2.63$ is 0.0001 and the expected number of deaths in one fire among 100 occupants is again 0.01.

The rationale behind the choice of k under a distributional assumption is that if the reliability of the estimation is poor, we cannot expect to estimate accurately the tails of the distributions. If the reliability is "excellent", the characteristic values will be taken as the 5th and 95th percentiles of the distribution. For a "good" reliability they will be taken as the 10th and 90th percentiles, for a "reasonable" reliability they will be taken as the 20th and 80th percentiles and for a "poor" reliability they will be taken as the 40th and 60th percentiles. The corresponding values of k for a lognormal distribution of X and Y are given in Table 1

Percentiles	k
5-95	1.64
10-90	1.29
20-80	0.84
40-60	0.25

Table 1: Values of k for a lognormal distribution of X and Y.

In addition, it should be pointed out that under the lognormal assumption the standard deviations σ_U and σ_V can be directly calculated from the mean and standard deviation of each of X and Y, using the well-known exact relations:

$$\sigma_U = \sqrt{\ln(1 + CV_X^2)} \quad (21)$$

$$\sigma_V = \sqrt{\ln(1 + CV_Y^2)} \quad (22)$$

where CV_X and CV_Y are the coefficients of variation of X and Y respectively.

These relations are however very robust and insensitive to distributional assumptions, particularly for small coefficients of variation.

5 Appendix: The lognormal distribution.

We say that X has the lognormal distribution if $U = \ln(X)$ is normally distributed. Suppose U has mean μ_U and standard deviation σ_U. Furthermore, denote the mean of X by μ_X and its coefficient of variation by CV_X. We then have the following two pairs of relations:

$$\mu_U = \ln\left(\frac{\mu_X}{\sqrt{1 + CV_X^2}}\right) \tag{23}$$

$$\sigma_U = \sqrt{\ln(1 + CV_X^2)}, \tag{24}$$

and reciprocally:

$$\mu_X = \exp(\mu_U + \frac{1}{2}\sigma_U^2) \tag{25}$$

$$CV_X = \sqrt{\exp(\sigma_U^2) - 1}. \tag{26}$$

References

[1] Melchers, R.E. (1987) *Structural Reliability: Analysis and Prediction.* Wiley.